Sixth Semester B.E. Degree Examination, Aug./Sept.2020
Finite Element Methods

Time: 3 hrs.   Max. Marks:100

Note: *Answer any FIVE full questions, selecting atleast TWO questions from each part.*

PART - A

1 a. Briefly explain the basic steps involved in FEM for stress analysis of elastic solid bodies.
                                                                             (10 Marks)
   b. Explain with a sketch Plane stress and Plane strain.   **(04 Marks)**
   c. Discuss the types of elements (1D, 2D, 3D) based on geometry.   **(06 Marks)**

2 a. Derive the stiffness matrix for 1 dimensional bar element using Direct Stiffness method.
                                                                             (08 Marks)
   b. Using Rayleigh – Ritz method, derive an expression for maximum deflection of the simply
      supported beam subjected to load 'P' as shown in fig.Q2(b). Use trigonometric function.
      Take E – Young's modulus I – Moment of inertia. A – Area of cross section of the beam.
                                                                             (12 Marks)

Fig. Q2(b)
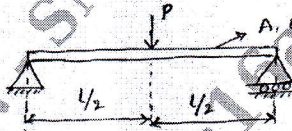
3 a. Derive the shape function for one dimensional bar element in Cartesian coordinates/global
      coordinate system.   **(10 Marks)**
   b. For the triangular element shown in fig. Q3(b) , obtain the strain displacement matrix 'B'
      and determine the strains ϵ_x , ϵ_y and γ_{xy} .
   Nodal displacements $\{q\} = \{2\ 1\ 1\ -4\ -3\ 7\} \times 10^{-2}$ mm. **(10 Marks)**

Fig. Q3(b)
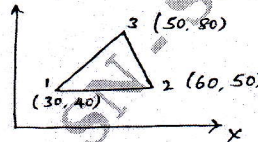
4 a. Solve the following system of simultaneous equation by Gaussian Elimination method.
      $x_1 - 2x_2 + 6x_3 = 0$
      $2x_1 + 2x_2 + 3x_3 = 3$
      $-x_1 + 3x_2 = 0.$ **(08 Marks)**
   b. Consider the bar shown in fig. Q4(b). An axial load $P = 200 \times 10^3$N is applied as shown.
      Using Penalty method, determine the following :
      i) Nodal displacement   ii) Stress in each material.   **(12 Marks)**

Fig. Q4(b)
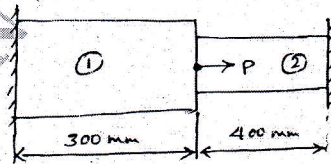
$A_1 = 2400$ mm^2
$E_1 = 70$ GPa
$A_2 = 600$ mm^2
$E_2 = 200$ GPa

PART - B

5 a. With neat sketches, define ISO, Sub and Super parametric elements. **(06 Marks)**

   b. Using two point Gaussian quadrature formula evaluate following integral.

$$I = \int_{-1}^{+1} \int_{-1}^{+1} (r^2 + 2rs + s^2)dr \cdot ds.$$ **(06 Marks)**

   c. Using Lagrangian method derive the shape function of a 3 – noded one dimension (1D) Bar element (Quadratic Bar element). **(08 Marks)**

6 a. Obtain an expression for stiffness matrix of a truss element. **(08 Marks)**

   b. For the two bar truss shown in fig.Q6(b), determine the nodal displacement and stress in each member. Also find support reaction. Take E = 200GPa. **(12 Marks)**
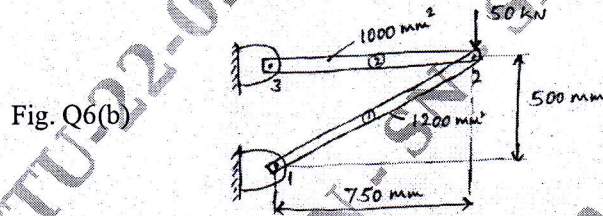
Fig. Q6(b)

7 a. Determine maximum deflection in the uniform cross section of cantilever beam shown in fig. Q7(a) by assuming beam as a single element. Take $E = 7 \times 10^9$ N/m^2, $I = 4 \times 10^{-4}$ m^4.
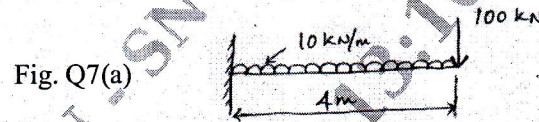
Fig. Q7(a) **(10 Marks)**

   b. A simply supported beam of span 6m and uniform flexural rigidity EI = 40000 KN-m^2 is subjected to clock wise couple of 300KN – m at a distance of 4m from left end as shown in fig. Q7(b). Find the deflection at the point of application of couple and internal loads.

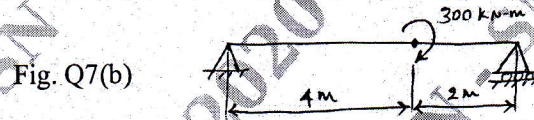Fig. Q7(b) **(10 Marks)**

8 a. Derive the Finite element equation for one dimension (1D) heat conduction with free end convection. **(08 Marks)**

   b. Determine the temperature distribution through the composite wall subjected to convection heat loss on the right side surface with convective heat transfer coefficient as shown in fig. Q8(b). The ambient temperature is -5^{0}C. **(12 Marks)**
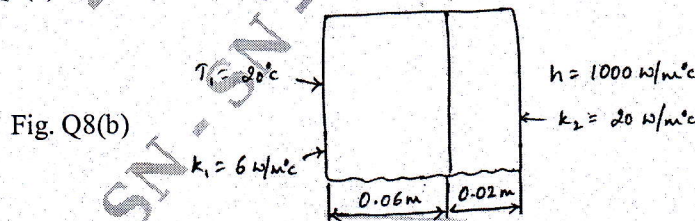
Fig. Q8(b)

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